On the structure of a laminar buoyant jet released horizontally

Sur la structure d’un jet flottant laminaire horizontal

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ABSTRACT

The flow generated by the injection of negatively buoyant fluid in water at rest was investigated by means of both the laser induced fluorescence (LIF) visualization and the particle tracking velocimetry. Statistics on series of LIF images acquired during the same experiment have been evaluated to give quantitative information about the localization of the structures. Such a phenomenon is controlled by two non-dimensional groups, the Reynolds and the Richardson number. Series of experiments have been carried out changing the Richardson number, whereas the Reynolds number was kept constant at a low value such that the initial flow was laminar. Visualizations permitted the identification of the structures that appear as a consequence of the instability of the flow. On the upper boundary Kelvin–Helmoltz instabilities occur that are very similar to the ones observed in a simple jet. The scenario on the lower boundary is completely different due to the unstable stratification. A descending plume develops as the distance from the outlet increases until it experiences transverse instability. It is very similar to the one generated by a cold cylinder in calm water. The presence of the plume generates perturbations that destabilize the upper boundary so that even a small amount of buoyancy is sufficient to considerably anticipate the apparition of Kelvin–Helmoltz instability and then the transition to turbulence of the flow. The velocity measurement permitted to quantitatively clarify that the buoyant jet is not axisymmetric. Its asymmetry decreases with the distance from the outlet. Evaluating the transverse velocity profiles it is possible to clearly see that the buoyant jet consists of two elements: a core characterized by high velocities and a surrounding zone, where the fluid moves slowly, that generates the downward plume.

RÉSUMÉ

L’écoulement généré par l’injection d’un fluide dense dans de l’eau au repos a été investigué à l’aide de la visualisation LIF (Laser Induced Fluorescence) et par PTV (Particle Tracking Velocimetry). Des statistiques sur des séries d’image LIF acquises durant la même expérience ont été évaluées pour donner des informations quantitatives concernant la localisation des structures. Un tel phénomène est contrôlé par deux nombres sans dimension: le nombre de Reynolds et le nombre de Richardson. Des séries d’expériences ont été réalisées en faisant varier le nombre de Richardson alors que le nombre de Reynolds est gardé fixe à une valeur suffisamment faible pour que l’écoulement initial soit laminaire. La visualisation a permis l’identification des structures qui apparaissent comme une conséquence de l’instabilité de l’écoulement. Des instabilités de Kelvin–Helmoltz se produisent à la limite supérieure, très similaires à celles observées dans un simple jet. Le scénario à la limite inférieure est totalement différent en raison d’une stratification instable. Une plume descendante se développe lorsque la distance à la sortie augmente jusqu’à l’apparition d’une instabilité transversale, très similaire à celle générée par un cylindre froid dans l’eau calme. La présence de la plume génère des perturbations qui déstabilisent la limite supérieure de telle sorte que si une petite différence de densité est suffisante pour anticiper considérablement l’apparition de l’instabilité de Kelvin–Helmoltz et par suite la transition vers la turbulence de l’écoulement. Les mesures de vitesses permettent de clarifier quantitativement que le jet n’a pas de symétrie axiale. Son asymétrie diminue avec la distance de sortie. En évaluant les profils de vitesse transversale il est possible de voir clairement que le jet est constitué de deux éléments: un cœur caractérisé par des vitesses élevées et une zone environnante, où le fluide bouge doucement, et qui génère une plume descendante.

Keywords: Buoyant jets, LIF, PTV, turbulence, mixing.

1 Introduction

Jets and plumes are phenomena that have been extensively investigated and a huge amount of literature have been produced about them. The former can be defined as the release of a fluid with a given velocity through an orifice in a field filled with receiving fluid that has the same characteristics as the fluid of the jet. The latter are due the release of potential energy that provides the fluid with positive or negative buoyancy relative to the ambient fluid. In the first case we can say that in correspondence to the source there is a flux of momentum, in the second case the motion of the fluid can be attributed to a source of buoyancy. These two phenomena can be regarded as two limit cases of a more general situation where both momentum and buoyancy are released at the same time: this phenomenon is called a buoyant jet. It happens when a fluid is released through an orifice that has a different density than the receiving fluid.
A buoyant jet has some peculiar characteristics that can be observed neither in a jet nor in a plume. If the ambient fluid is at rest, both jets and plumes have an axisymmetric structure; conversely a buoyant jet conserves symmetry only as long as it is released vertically, so that momentum and buoyancy act in the same direction. Most literature on buoyant jets focuses on this case, since its symmetry properties make the treatment of the problem much easier (Papanicolaou and List, 1988; Papantoniou and List, 1989; Pantokratoras, 2001). Some consider the general case but they do not focus on the evaluation of the physical mechanisms that cause the asymmetry of the jet structure and contribute to the bending of the axis trajectory (Cederwall, 1967; Fan, 1967; Hansen and Schroder, 1968; Anwar, 1972). In addition, due to its practical interest in ocean outfalls and other applications, they consider the case of turbulent flow at the outlet.

Other authors predicted the behaviour of the buoyant jet by using integral equations, formulated under the assumption of axial symmetry and self-similar transverse profiles (Fisher et al., 1979; Wood et al., 1993; Pantokratoras, 1998). These models are very useful for engineering purposes such as environmental assessment, but do not help to clarify the transverse structure of the buoyant jet. Few consider laminar initial conditions, and non-vertical release, but do not provide information on the velocity field (Sarynarayana and Jaluria, 1984; Arakerti et al., 2000). As a consequence, the main questions on the structure of a buoyant jet released horizontally are still open.

In the case of stagnant receiving fluid, the only parameters that characterize the boundary conditions are the initial fluxes of volume and momentum and buoyancy per unit mass, that hereafter are indicated with $Q$, $M$ and $B$, respectively (Fisher et al., 1979):

$$ Q = \int_{S_0} u \, dS \quad M = \int_{S_0} u^2 \, dS, \quad B = \int_{S_0} \frac{\Delta \rho}{\rho} \, u \, dS $$

where $S_0$ is the area of the orifice and $u$ the fluid velocity. From these parameters two length scales can be obtained:

$$ \begin{cases} 
I_Q = \frac{Q}{\sqrt{M}} \\
I_M = \frac{M^{3/4}}{B^{1/2}} 
\end{cases} \tag{1} $$

The former length scale is based only on the jet-like properties. As a matter of fact it describes only the exit geometry ($I_Q = \sqrt{S_0}$), so if the distance from the orifice, $s$, is of order $I_Q$, the initial conditions are meaningful, and for $s \gg I_Q$ they do not influence the flow anymore and the jet is fully developed. The latter accounts for the effects of buoyancy and can be proven, by considering the limiting solutions, that for $s \ll I_M$ the buoyant jet behaves as a jet, and for $s \gg I_Q$ it behaves like a plume. The ratio between the two length scales represents the relative importance of buoyancy and momentum and is called the Richardson number:

$$ Ri = \frac{I_Q}{I_M} = \frac{\sqrt{B}}{\sqrt{M^5}} \tag{2} $$

that is the first non-dimensional group controlling the flow. Some authors prefer to describe the flow in terms of densimetric Froude number,

$$ Fr = \frac{U}{\sqrt{\frac{\rho \Delta \rho}{\rho_D} g}} $$

with $D$ the diameter of the outlet and $U = Q/S_0$ the mean initial velocity. The two parameters are related by the equation:

$$ Ri = \frac{\sqrt{\pi}}{4} \cdot \frac{1}{Fr} $$

Here the Richardson number is preferred since the case of a simple jet, a real case, gives an infinite Froude number, a limit case, whereas an infinite density difference between jet and ambient fluid (a limit case) gives $Fr = 0$. On the contrary $Ri = 0$ for zero density difference and infinite when the density difference is infinite.

In addition, by including viscosity into the set of relevant parameters, one obtains the Reynolds number:

$$ Re = \sqrt{\frac{M}{\nu}} $$

that is related to the usual definition of Reynolds number in terms of diameter and mean velocity, $Re^* = UD/\nu$: $Re = \beta \sqrt{\frac{M}{\nu}} Re^*$; where $\beta$ is a coefficient that accounts for the shape of the velocity profile. It is $\beta = 1$ for a flat profile (e.g. turbulent) and $\beta = \frac{1}{4/3} \approx 1.15$ for a parabolic profile. This is the second non-dimensional group that one should take into account. Finally, if the jet is aimed at an angle, $\theta$, to the vertical, this is the third non-dimensional parameter influencing the flow.

On the basis of the above dimensional analysis and assuming self-similarity both the jet and the plume behaviour can be described. Under the same assumptions a buoyant jet released vertically can be described in terms of simple jet behaviour at distances much lower than $I_M$ and in terms of plume behaviour when the distance, $s$, is much greater than $I_M$.

Nevertheless, these results are not extendable to the general case of a non-vertical buoyant jet since the flow is neither axisymmetrical nor self-similar: when the buoyant fluid is released non-vertically, the jet bends upwards or downwards depending whether the fluid released is lighter or heavier than the receiving one: it results in a curved jet that looks similar to a jet in cross flow. Nevertheless, the mechanism that determines the curvature of its axis is completely different. Sometimes an analogy is used to describe jets in cross flow: close to the outlet the jets behaves as a cylinder immersed in a constant flow. In the same way a buoyant jet can be described as a hot cylinder immersed in a calm fluid and it will be shown that the two flows exhibit many similar features.

Assume that the jet is discharged horizontally and that the released fluid is heavier than the receiving one. Then one finds a stable stratification at the upper boundary where the heavier fluid of the jet is under the lighter receiving fluid, and an unstable stratification at the lower boundary where the fluid of the jet is above the less dense receiving fluid. The different stability conditions determine very different behaviour at the two opposite sides. Where the stratification is stable the mixing between the jet
and the ambient fluid is inhibited and the entrainment is low; on the opposite side the unstable stratification enhances the mixing and thus the entrainment is much higher. Moreover, the nature of the coherent structures that arise from the instability of the jet is very different on the upper and lower side. These differences between the opposite sides of the jet are the mainly responsible for the curvature of its axis.

In order to clarify the behaviour of such a flow, and the role of the different kind of instabilities that take place, an horizontal buoyant jet has been generated. The flow at the outlet was laminar so that the initial velocity profile (that is known to be a crucial factor in deciding the behaviour of the flow) was parabolic (Hussain and Zaman, 1981). The Reynolds number was fixed, whereas the Richardson number was varied by changing the density difference between the released fluid and the receiving one. Visualizations by laser induced fluorescence (LIF) have been used to investigate both the structure of the flow both in the longitudinal and in the cross-sectional plane, quantitative observations of the velocity field in the longitudinal plane have been carried out by means of particle tracking velocimetry (PTV). Series of experiments have been realized by releasing a fluid with different density into a glass tank filled with water. The released fluid also was water and its density was changed by adding salt or varying its temperature.

2 Experimental set-up

Experiments were carried out in a glass tank 0.80 m high, 0.80 m wide and 1.32 m long. The jet was discharged through a smooth pipe, 0.012 m in diameter. The jet was supplied by a constant head tank. The water level in the main tank was kept constant by letting the excess water flow out through a gutter. With the fluid released is brine, after few minutes an increasing layer of heavy water lay on the bottom of the tank. As the top of this layer reaches the investigation area the experiment must end. To slow the growth of this layer an additional wall was placed on the side of the gutter, 0.05 m high on the bottom of the tank. As a consequence, the excess water flowing out of the tank was kept from the lower, stratified layer rather than from the surface (see Fig. 1).

To let the laminar flow fully develop before the exit section the pipe releasing the jet was 2.0 m long (corresponding to more than 166 diameters). Disturbances due to the flow were minimized by means of a chamber with a convergent section placed just before the straight pipe. A sketch of the experimental set-up is shown in Fig. 1.

As mentioned above, the working fluid was water; depending on the required density difference, salt was added to the released fluid, or a temperature difference between receiving and discharged water was imposed.

During visualization experiments fluoresceine was added to the buoyant jet and the investigated plane was illuminated by a light sheet generated, through a cylindrical lens, by a 4 W Argon-Ion Laser. A video-camera, orthogonal to the illuminated plane, recorded the images. Visualization has been performed both in the longitudinal, vertical plane containing the axis of the jet, and in series of planes, orthogonal to the axis, placed at different distances from the outlet.

Velocity fields have been measured by means of PTV (Cenedese and Querzoli, 2000). During these experiments the discharged fluid was seeded with non-buoyant pollen particles, 50 µm in diameter. Illumination and image recording equipment was the same used for the LIF experiments. The measuring plane was illuminated by a laser-light sheet and the video-camera recorded the series of images. Images were successively digitized at 720 × 576 pixels resolution and analysed in order to identify the particle locations.

According to the PAL standard, frames are generated at a 25 Hz frequency and each frame consists of two fields, acquired at 1/50 s time lag, that correspond to the even and odd rows of the frame. During the successive image analysis even and odd rows were considered separately so that particle locations were known at a 50 Hz frequency.

Particles were recognized by thresholding fields and individuating the lit connected areas. Their locations were identified by computing an average weight of the grey level of the pixels exceeding the threshold.

Particle locations at the different times were analysed in order to recognize their trajectories. The tracking procedure was based on two basic parameters: maximum distance between two successive locations of the same particle (corresponding to a maximum admissible velocity) and maximum difference between two successive displacements of the same particle (corresponding to a maximum admissible acceleration). When more than one particle matched the second condition at a given time, a minimum acceleration criterion was adopted. Parameters were adjusted in order to reduce the number of cases of ambiguity to less than 1% of the number of applications of the above rule. Velocities have been obtained from the trajectories by dividing particle displacements by the time interval.

3 The set of experiments

The phenomenon under investigation is influenced by three dimensionless parameters, and an exhaustive description of the
dependence on every one of them would require a huge set of experiments. As a consequence, there are some choices that must be made in order to delimit the field of investigation. The angle \( \theta \) influences the initial angle between momentum and buoyancy; it ranges from 0 to \( \pi \). The value \( \theta = \pi / 2 \) chosen in this experiment can be considered a limit case with the maximum effect of the non-alignment between momentum and buoyancy on the flow.

Provided that the pipe discharging the fluid is long enough, the Reynolds number completely determines the characteristics of the flow at the exit. For low Re the velocity profile is parabolic and the flow laminar; increasing the Re leads to a transition to turbulence; finally, for high enough values of Re a flat turbulent profile is obtained. Between these three regimes, the first is chosen in the present experiment for the investigation. An arbitrary, fixed value of Re have been chosen that ensure a laminar regime is obtained. Varying Re within the range of laminar flow would not change the nature of the phenomenon but only the relative relevance of the viscosity in the flow after the exit (Abramovich and Solan, 1973). As a consequence only one value of Re, inside the laminar range, have been used during the experiments.

A buoyant jet can be regarded as a flow generated by two concurrent energy sources: the flow is supplied with kinetic energy through the momentum flux and with potential energy through the buoyancy flux. The third parameter, that is, Ri, describes the relative importance of buoyancy and momentum. Changing this parameter would change continuously the nature of the buoyant jet from a simple jet (Ri = 0) to a plume (Ri \( \to \infty \)). The dependence from this dimensionless parameter was studied in order to understand the phenomenon.

Some preliminary experiments were performed in order to verify the initial condition for the jet. Transverse profiles of axial velocity have been measured immediately after the outlet by means of laser Doppler velocimetry (LDA) at different Reynolds numbers (Fig. 2). The effluent fluid was fresh water with no density difference with the ambient water since the buoyancy flux does not produce meaningful effects in the proximity of the discharge. The profiles are observed to be closely parabolic up to a Re = 1900, indicating the flow is laminar and fully developed; whereas for larger Re the profile is clearly turbulent. As a fixed value, well within the laminar regime range, Re = 1100 have been chosen. The set of values used to explore the dependency of the phenomenon on Ri is reported in Table 1, together with the other characteristic parameters of the experiments.

In order to characterize the structure of the buoyant jet, both visualization by LIF and quantitative measurements by PTV have been performed in the longitudinal plane. In addition, for three Richardson numbers, Ri = 0.05, 0.13 and 0.22, series of transverse visualizations have been performed. LIF visualization were carried on also for Ri = 0 (simple jet) but the result have not presented here except for some information used for comparison since the phenomenon have been extensively investigated in the past (Rankin et al., 1983; Cohen and Wygnanski, 1987).

### 4 Results and discussion

#### 4.1 General structure of the buoyant jet

Instantaneous LIF pictures can be usefully chosen as a starting point in the description of the structure of the buoyant jet since they show qualitatively all the main phenomena involved in its evolution. Figure 3 shows two pictures at Ri = 0.05 that represents a vertical plane containing the axis of the jet and four transverse visualizations on the planes drawn in the long field longitudinal picture. This is the experiment with the lower buoyancy effect and the laminar jet can develop from the outlet without destructive instabilities for a long tract. Anyway, the presence of buoyancy has the effect to bend downwards the jet. Looking at the transverse sections corresponding to this tract (a and b), the development of a vertical plume is observed. In section a there is only a deformation of the, initially round, cross-shape of the jet, then a real plume takes place and begins to bend transversally as it begins to be unstable. In section b it is shown during one of these bendings that are very slow and apparently non-periodic. Moreover, once the plume has assumed one curved configuration can maintain it substantially unchanged even for a long time. In that condition, the structure can be still considered marginally stable.

To explain the origin of the downward plume let imagine that the source of buoyancy is the temperature. In the first part of its development, before that any kind of instability has time to take place, the effect of the presence of the jet is to have a cold, cylindrical region, surrounded by calm water. Therefore, one can think of the core of the jet as equivalent to a cold cylinder placed in the

![Figure 2](image-url)
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Recipient water at rest. The section at the outlet corresponds to a condition in which the cooling has had no time to act on the flow, and there is a round core surrounded by calm water. Increasing the distance from the outlet corresponds to considering sections where the cooling have had more and more time to generate the downward plume. Following this similarity, the plume observed in the buoyant jet is equivalent to the one generated by an horizontal cylinder instantaneously cooled. The distance from the outlet is equivalent to the time passed since the start of the cooling of the cylinder.

After the phase of development of the plume, two kinds of instabilities take place: on the upper side, the structure is substantially similar to the one of a simple jet, with strong shear between the zone of the core and the ambient; as a consequence the well known Kelvin–Helmoltz instability occurs (Drazin, 1970). An example of these structures can be observed in the close field longitudinal visualization. On the lower side, there is the above described plume, that grows until it give rise to the transverse instabilities, similar to those generated in the plume of a cooled cylinder. They propagate from the core of the jet to the tip. Their presence is indicated both in the close field, longitudinal view and in the section c. In the same section, a secondary instability, similar to the one observed around a simple jet, can be noticed as well.

Figure 3 Instantaneous flow visualizations at Ri = 0.05. Transverse visualizations at the following distances from the outlet: s/D = 12.5, 17, 21 and 25.

After the full development of the Kelvin–Helmoltz and plume instabilities, a sudden transition to turbulence occurs. The transition rapidly involves both the upper and the lower part of the jet (section d). Even if it has been observed that the jet is not axisymmetrical even after the transition, as long as its axis is curved, the present paper focuses only on the near field, until its transition to turbulence.

In summary, we can explain the structure of the buoyant jet in terms of an upper part behaving as a simple jet and a lower part in which a plume is generated by the cool core of the jet. This plume is similar to the one due to a cold cylinder. Both upper and lower flows can experience instabilities that generate vortical structures. The interaction of these structures determines the transition to turbulence.

4.2 Effects of the Richardson number

Once the elements that contribute determining the evolution of the buoyant jet have been identified by examining the visualization at Ri = 0.05, the dependence of the behaviour of the flow on the Richardson number can be educed by comparing visualizations at different Richardson numbers.

Figure 4 shows longitudinal and transversal visualizations at Ri = 0.13. In this experiment the influence of the buoyancy is
more intense and this is made evident by the higher curvature of
the jet axis and by the fast development of the lower plume—
as a matter of fact, there is not a meaningful almost cylindrical
tract of the jet. Consequently, the transverse instability of the
plume appears closer to the outlet than in the first experiment.
At Ri = 0.13 the plume drives the Kelvin–Helmoltz instability
on the upper side. There are two observations that support this
assertion: firstly the Kelvin–Helmoltz instability appears well
before that in the first experiment even if it should be driven
only by the Reynolds number and not by the buoyancy; sec-
ondly, the close view picture on the upper-right of Fig. 4 clearly
shows clearly that plume instabilities and Kelvin–Helmoltz ones
are coupled. Since plume instability occurs before the Kelvin–
Helmoltz one it is reasonable to suppose that its is the former
to drive the latter. Looking at section c, it may be noticed that
the plume becomes turbulent at its lower end. This is due to the
fact that in this experiment the action of the buoyancy is more
intense, therefore the development of the plume is more rapid
than the one of the upper “jet-like” side. As a consequence the
transition of the plume occurs before the whole jet get turbulent.
As a consequence, the turbulence appears first at the lower and
of the plume, without inducing any transition into the core and
only later in the core, driven by the Kelvin–Helmoltz instability.

The same picture also exhibits a secondary instability develop-
ning at the round core of the jet. These structures have been
observed to interact with the plume instabilities, propagating
downwards.

In the early stages of development (sections a and b of Fig. 4)
an additional phenomenon can be observed: on the side of the
plume there are two small tongues of dye propagating down-
wards. They develop more or less at the same time as the main
plume but are not seen anymore at the later stages (sections c
and d). Using once again the similarity of the cooled cylinder
to explain the phenomenon, they may be thought of as corre-
sponding to the detachment of the flow from the ideal cylinder.
Moreover, they are seen to disappear as the plume instabilities
rise. Anyway a clear explanation cannot be proposed without a
more extensive investigation.

In Fig. 4 the visualizations of an experiment at Ri = 0.22
are reported. In this experiment the sequence of appearance
of the turbulence depicted above is more enhanced. The tur-
bulence appears at the lower end of the plume well before the
Kelvin–Helmoltz structures appear (sections a and b and lon-
gitudinal visualizations). As a consequence turbulence appears
first at the lower end of the plume, then propagates upwards
as the plume becomes more unstable; finally it reaches the
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Figure 5  Instantaneous visualisations at Ri = 0.22. Transverse visualizations at the following distances from the outlet: s/D = 4, 5, 6 and 8.

Figure 6  Instantaneous visualisations at Ri = 0.28.

core of the jet. Due to the early instability of the plume, the Kelvin–Helmoltz structures have not time to develop completely and to give rise to their transition to turbulence before they are destroyed by the turbulence generated by the plume instability.

The same kind of behaviour is shown in the pictures at Ri = 0.28 (Fig. 6). The plume starts it development after less than one diameter from the outlet, and in a few diameters it reaches its instability at the lower end. Even in this case it is the plume that drives the generation of turbulence.
4.3 LIF statistics

The instantaneous pictures that have been described above are very useful for the individuation and the comprehension of the structures of the flow, nevertheless, the comparison of a structure or the transition to turbulence at a given location is an accidental event, and cannot be taken as a rule for the investigated flow, therefore it is not possible to extract from one of them quantitative information about the general behaviour of the flow. To understand what happen on average, it is more useful to compute some statistics on the LIF images.

From the theoretical point of view, one could state a calibration law that relates the light intensity measured on a picture with the concentration of fluoresceine in any location of the field. As a consequence, one could consider the LIF technique as a method to measure the behaviour of a passive scalar. Anyway, such an approach requires very careful attention to be paid to a lot of details: first, the light energy emitted from a portion of fluid depends both on the concentration of fluoresceine and on the amount of light impinging on the region. In turn, the amount of impinging light depends on the distribution of the light on space from the source (e.g. a laser beam spread with a cylindrical lens has likely a Gaussian distribution) and on the portion adsorbed during its path, that is related to the concentration of fluoresceine along the optical path preceding the illuminated location. This effect is not meaningful only if the concentration of fluoresceine is very low. Secondly, it must be considered the response of the CCD of the video-camera that is not linear and suffers phenomena, such as the saturation, that makes impossible, for certain light ranges, to state an unique relation between measurement and concentration. Finally, there are other accidental factors such as the non-uniform adsorption of the glass wall of the tank, impurity in the water, undesired reflections and so on. After one accounts for all of these factors, the uncertainty is so high that it is difficult to consider the measurement reliable. For that reason it has been chosen to not consider any relation between light intensity and concentration. Nevertheless, many interesting conclusions can be deduced from the evaluation of the statistics of the LIF images.

In particular, statistics presented include in Fig. 7 the mean light intensity, in Fig. 8 its variance, and in Fig. 10 the fluctuation intensity, that is defined here as the variance of the light intensity divided by its local mean value. The latter gives an idea of the importance of the fluctuations of light intensity at a given location relative to the mean value. In that way one can observe clearly also small fluctuations in zones where the mean light intensity is low (e.g. at the borders of the jet). Statistics have been computed over an ensemble of 1500 images taken at 12.5 Hz frequency, corresponding to a total acquisition time of 120 s. Statistics are presented for Ri = 0.05, 0.13, 0.22 and 0.28.

Looking at the mean fields, the increase in the jet curvature with the Richardson number is apparent (Fig. 7). At Ri = 0.05 it is confirmed that the jet remains a stable and round cylinder until about 19 diameters. Then turbulence appears and the jet suddenly increases in diameter. According with the different behaviour described, the spreading of the jet is different on the upper and lower side, being larger at the bottom. This different growth rate contributes to the axis curvature. It is also apparent that the axis of the buoyant jet is also curved in the stable zone, therefore the curvature of the axis in the turbulent zone is the result of two contributions: the first is the higher density of the released fluid, that acts both in the laminar and in the turbulent zone, the second is due to the different development of the upper and lower side of the jet in the turbulent zone. At the same time the maximum grey level (i.e. concentration) decreases abruptly when turbulence appears due to the spreading of the flow and to the intense entrainment.

At higher Richardson number the buoyant jet is much more curved downwards and the different structure of the upper and lower side appears clearly. Though the jet enlarges more rapidly on the lower side, the maximum grey level remains close to the upper boundary, as a consequence the cross profiles mean concentration are everywhere skewed; the skewness of the distribution decreases as the distance from the outlet increases. In fact the distribution has been observed to be nearly Gaussian in the vertical tract of the jet at Ri = 0.22 and 0.28.

In particular at Ri = 0.13 and 0.22, the appearance of Kelvin–Helmoltz structures always at the same location generates a meaningful deformation of the mean jet. Looking, for example, at the experiment with Ri = 0.22, the upward translation of the maximum is apparent in correspondence with the place where the structures appear (indicated in Fig. 7 with K–H).

The transition to turbulence is clearly indicated by the variance of the luminosity of the LIF images that is drawn for different Richardson numbers in Fig. 8. It is presented in arbitrary units not indicated in the plots. As a matter of fact, the variance is very low in the zone of the jet that precedes the arising of the structures and it is a maximum in the zones where the Kelvin–Helmoltz and plume instabilities grow. Finally a sudden increase in the variance of the whole jet is observed that corresponds to the transition to turbulence. The plots of Fig. 8 have been analysed in order to identify the transition distance from the outlet in the set of experiments. As shown in Fig. 9, the transition is observed to occur at a distance that is linearly proportional to the length scale \( l_M \); the coefficient of proportionality is very close to the unity (1.01). It is worthwhile to notice that this linear relation cannot be generalized as \( l_M \) tends to infinity for a simple jet, since the transition occurs at a finite distance from the outlet (transition was localised at 34 diameters in a simple jet with Re = 1100 generated in the present tank). Anyway the high linearity of the dependence on the buoyant length scale, \( l_M \), suggests that, within the investigated range of Richardson number, the transition is basically driven by the buoyancy. Moreover, as the ratio \( l_M / D \) is related to the Froude number by a multiplicative constant, it may be also concluded that the distance of transition is linearly proportional to the Froude number (and inversely proportional to the Richardson number) in the investigated range.

In Fig. 10 the fluctuation intensity is presented for the set of LIF experiments. Relative fluctuations of luminosity are highly correlated to the presence of coherent structures. As a matter of fact, both Kelvin–Helmoltz and plume instabilities are
characterized by high luminosity gradients (see visualizations of Figs 3–6); as structures move, the relative fluctuations are high. By contrast, in the zones where high turbulence levels are present, the intense mixing abates the fluoresceine concentration gradients and luminosity fluctuations. Therefore, the zones where instabilities appear and grow are clearly identifiable in Fig. 10. Comparing, for example, the plot corresponding to $\text{Ri} = 0.22$ with the mean value one of Fig. 7, it is confirmed that the deformation in the mean value with the upward translation of the maximum value zone is due to the appearance of the Kelvin–Helmoltz structures. In addition it may be noticed that, according to the discussion about the instantaneous visualizations, the experiment at $\text{Ri} = 0.05$ is the only one where the Kelvin–Helmoltz structures appear before the plume becomes unstable. In all the other experiments the plume instabilities appear before and drive the transition to turbulence.

4.4 Analysis of LIF images sequences

The effect of the buoyancy on the stability of the jet can also be investigated by evaluating the kinematics of the structures. Motion of coherent structures can be quantitatively identified by analysing series of LIF images (i.e. movies). In particular, the quantities that have been measured are the frequency of appearance of the Kelvin–Helmoltz structures and the velocity of the transverse waves that characterize the instability of the plume.

The former measure has been performed by considering the longitudinal visualizations, and focusing on a transverse section placed at the distance from the outlet where the Kelvin–Helmoltz structures are completely formed. On that section, the transit of a structure corresponds to a sudden widening of the jet. Therefore, the frequency of arrival could be obtained through a Fourier analysis of the time history of the upper boundary of the jet. Results are shown in Table 2 as a function of the Richardson number. The frequencies have been non-dimensionalized to give the Strouhal number:

$$\text{St} = \frac{f \cdot \delta}{U}$$

where $f$ is the investigated frequency and $\delta$ is the momentum thickness (for a parabolic velocity profile $\delta = D/15$).
According to the above discussion, the buoyancy tends to destabilize the jet, and the frequency increases with the Richardson number. The trend seems to be almost linear in the range of Ri investigated even if five points are not enough to draw a reliable conclusion. This is consistent with the value of the Strouhal number of Kelvin–Helmoltz structures that has been measured for a simple jet, St = 0.014 (Husain and Hussain, 1983), as it is of the same order of magnitude of the additive constant in the linear approximation equation.

The velocity of the perturbations along the plume was measured by considering a vertical section of the images of the longitudinal visualization movies, corresponding to the zone where the downward moving waves were clearly visible. On the line of the section, the waves appeared as dark and bright zones moving down. Then, the time history of the luminosity along the

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**Table 2** Strouhal number, St, and non-dimensional velocity of propagation of Kelvin-Helmoltz structures, vke/U, at different Richardson numbers

<table>
<thead>
<tr>
<th>Ri</th>
<th>St</th>
<th>vke/U</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>0.13</td>
<td>0.039</td>
<td>0.131</td>
</tr>
<tr>
<td>0.22</td>
<td>0.051</td>
<td>0.197</td>
</tr>
<tr>
<td>0.28</td>
<td>0.063</td>
<td>0.361</td>
</tr>
</tbody>
</table>
section was used to compute the time–space correlation. This is a two-dimensional plot that exhibits a maximum along a direction that represents the velocity of the dark and bright zones, that is, of the waves.

Results of the above described procedure are presented in Table 2 versus the Richardson number. The experiment at \( R_i = 0.05 \) did not furnish any result since the decaying of the Kelvin–Helmoltz structures occurs before the plume instabilities can develop. Three values are indeed not enough for a general law to be extrapolated, nevertheless, it is clear that the velocity of the waves increases with \( R_i \).

4.5 Velocity statistics

A set of experiments was performed in order to investigate the velocity field. During these experiments the jet fluid was seeded and the velocity of the particles was computed through the PTV technique described above. Since in this kind of velocimetry the samples are obtained at random locations where seeding particles are found, an interpolation is required in order to obtain the velocity statistics on a regular grid. To this end, the investigation area was divided into \( 40 \times 20 \) rectangular sub-regions and the mean of the velocity samples on each region was computed and reported at the node in its centre. This is a rather rough method of approximation but has the advantage of not inserting any artificial spatial correlation or artificial smoothing of the results since the value at each node of the grid depends only on the samples in the corresponding sub-region.

In the the plots of Fig. 11 the mean velocity field is represented for each experiment. The background grey level indicates the magnitude of the velocity vectors, \( \vec{v} \), non-dimensionalised by means of the mean velocity at the outlet, \( U \). From those fields the jet axes have been computed by identifying the set of points where the velocity is a maximum. The corresponding lines are drawn in white. In addition at each Richardson number, some transverse profiles of mean velocity are plotted (Fig. 12). The lines on which the profiles are computed are drawn in black on the mean velocity field plots of Fig. 11.

The mean velocity fields clearly show the structure of the buoyant jet: the axis is curved downwards and the curvature is increasing with the Richardson number. The maximum values of the velocity are measured near the outlet, with values a little lower than twice the mean outlet velocity, accordingly to the properties of the laminar, parabolic profile at the exit. The velocity profiles are skewed with a long tail on the lower side. The skewness is more evident in the curved part before the transition to turbulence and tend to diminish as the distance from the outlet increases, in particular after the turbulent transition (in agreement with the
central limit theorem). The transverse structure of the part of the jet that is above the axis is quite different from the one below.

Above the axis there is the zone where it has already been recognized that the structure is similar to that of a simple jet: near the outlet there is sharp interface between the jet and the ambient, characterized by high velocity gradients and a velocity profile that changes shape from parabolic to Gaussian in a few diameters from the outlet (see transverse profiles, Fig. 12). In that zone the width of the jet increases moderately. After transition to turbulence, the growth rate is enhanced and the velocity gradients are smoothed correspondingly.

The part of the jet below the axis exhibits a substantially different behaviour. This is the part where the plume-like structure develops. Focusing, for example, on the transverse profiles at $s/D = 2, 4$ and $6$ at $R_i = 0.11$, it is apparent that for positive values of the radial abscissa, $r/D$, the velocity profile can be

Figure 11  Mean velocity field at different Richardson numbers. Grey scale represents the dimensionless magnitude of the velocity module, $v/U$; white line indicates the jet axis and the black lines the transverse sections of Fig. 12.
On the structure of a laminar buoyant jet released horizontally

Figure 12 Transverse profiles of mean velocity magnitude non-dimensionalized by means of the mean exit velocity $U$. Distances from the outlet are indicated in diameters in the top right box.

divided into two parts: close to the axis there is a zone with high velocity and velocity gradients, then there is a zone where the velocity gradient decreases sharply and the velocity is low. A similar behaviour is observed in the transverse profiles at $R_i = 0.13$, $0.22$ and $0.31$, whereas it is not apparent at $R_i = 0.05$ since the turbulent transition occurs before the plume-like structure can develop.

This structure is consistent with the scheme suggested by Arakeri et al. (2000) in which the buoyant jet is divided into a core, characterized by high velocities and an external zone with low velocities. As a matter of fact, the above described structure of the profiles, preceding the turbulent transition, gives the evidence that the two zones actually exist and are clearly separate. The core includes all of the profile above the axis and the zone below where the velocity is high. Its width is about one diameter and does not increase meaningfully before the transition. The external zone includes the lower part of the profile, after the sharp change in the velocity gradient. Observing the velocity fields at the various $R_i$, it may be noticed that this structure begins at the outlet and develops until the turbulent transition occurs; then the turbulent mixing destroys it in a few diameters.

The overall shape of the profile in the core is roughly Gaussian and symmetric: the skewness of the profile depends almost entirely on the presence of the plume.

The velocity measurements also permit to depict some of the principal characteristics of the plume-like zone. First of all it should be noticed that here the fluid motion is substantially slower than in the core. This zone consists partially of the fluid at the
The interactions seem to be mainly due to the instabilities that by MIUR and CNR. This work was supported to the experimental work. The evaluation of the results suggests the idea that the development of the plume-like zone is quite independent from the core. The interactions seems to be mainly due to the instabilities that determine perturbations from the external zone to the core and vice versa. As a consequence, the development of the buoyant jet is controlled by the Richardson and Reynolds number mainly for the influence that they have on the appearance of instabilities and on the transition to turbulence. In particular it is apparent that the buoyancy, even when it is very weak, plays a fundamental role in the transition to turbulence of the jet. As a matter of fact, the first comparison of the Kelvin–Helmoltz instabilities in an experiment without buoyancy, but at the same Reynolds number of the others presented, was observed at 34 diameters from the outlet. The addition of a small amount of buoyancy (\( \text{Re} = 0.05 \)), induce a large upwind translation of the comparison of these instabilities. It occurs at about 19 diameters, with a decrease of 15 diameters. This means that the Kelvin–Helmoltz structures are excited by perturbations due to the buoyancy, perhaps due to some starting instabilities of the lower plume. A further increase in the buoyancy does change the order in which the different parts of the buoyant jet become unstable; anyway the net effect is to reduce furthermore the distance at which instabilities appear.

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References

